Application of mathematical modelling of Screw Engins to the optimization of lobe profiles

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Abstract

A differential mathematical model of the thermodynamic process in a screw engine, incorporated into the computational package SCORPATH (Screw Compressor Optimal Rotor Profiling and THermodynamics) together with a general specification of the screw geometry, was employed to improve the engine performances by the optimization of the lobe profiles. A new profile of a slender shape with thinner lobes in both the main and gate rotor is considered, which yields a larger cross-sectional area and shorter sealing lines resulting in higher delivery rates for the same tip speed. The paper discusses some modelling issues and focuses on the optimization of the gate rotor tip radius as an example which demonstrates one of many possible applications of the mathematical modelling to the optimization of design and operating condition of a screw engine.

Introduction

Screw engines find nowadays a diversity of application as compressors or expanders, operating dry, oil flooded or with other fluids injected in engines during the compression or expansion process. A variety of working fluids are used, ranging from air and common gases to new refrigerants involving complex two-phase flows and phase changes within the engine. Each of the cases mentioned, as well as many subvariants, usually require a specific design, as well as operating regime in order to achieve optimum performances. Hence, neither universal rotor configuration nor set of the working parameters can be defined even for a narrow class of the engines. Because of geometric complexity and a variety of applications, a closer insight into the thermodynamic process and an analysis of the influence of various design parameters on the engine performances are probably more important, though less straightforward, than in other types of engines which perform similar functions. For that reasons a need for an optimization tool and well defined criteria is a prerequisite for achieving optimum performances for each specific application, for further improvement of screw engines design, as well as and for broadening their application versatility.

A major break-through in the profile development was the replacement of symmetric profiles by the asymmetric SRM shape where a system of cycloides was introduced in the high-pressure side of the lobes instead of circles. This led to a substantial reduction in the blow-hole area, and a corresponding increase in delivery rate and efficiency. The front side of the teeth was additionally modified through the eccentricity of the gate rotor circle, which resulted in different corresponding shape of the lobe front of the main rotor. The resulting SRM "C" profile emerged as the most successful twin screw profile and is still widely used for various applications. After years of intensive use, it has been recognized, however, that further improvements are possible and desirable and in some applications the "C" profile has been recently substituted by some new, more efficient lobe shapes.

A major incentive for an intensified search for new profiles came from recent development of the mathematical modelling and computer optimization. In fact, the approach to the optimum design of the screw rotors lobe profiles has substantially evolved over the past few years through a more extensive use of computer modelling and it is likely to lead in the near future towards further improvement of the engine performances. Unlike in the past, when the improvements were achieved by lucid intuition and tedious trial-and-error tests, the computer modelling offers nowadays a powerful tool for process analysis and optimization. However, the complex process, as well as geometry, requires that various physical phenomena be modelled in an approximate manner, leaving many possibilities for modelling improvements. Computer models and numerical codes, reported in open literature, differ between each other often in the approach and even more in mathematical level at which various phenomena are modelled. A lack of comparative experimental verification hinders still a comprehensive validation of modelling concepts, but in spite of that, the modelling and computer optimization are steadily gaining in credibility and are being more and more employed for design improvement.

The present paper describes some results of a recent effort to improve the computational model of the process in a screw engine. It report also on the model application to the development of a new profile, with thinner lobes in both the main and gate rotors, yielding larger cross section areas and enabling higher delivery rates for the same tip speeds. The new profile has also shorter sealing lines resulting in lower leakages. The paper describes the modelling background, discusses the rationale behind the new lobe shapes and focuses on the influence of one of the major lobe parameters - the gate rotor tip radius and its optimization.

Modelling Rationale

The effects of profile modifications was investigated by employing a computational model, based on conservation equations for mass and energy in a differential form, which describe the thermodynamic and flow process in a screw engine in terms of the rotational angle (or time). The model also incorporates the differential kinematic relationships which define the instantaneous operating volume and its change with the rotation angle. This enables the computation of the evolution of all thermodynamic and fluid properties in time within the engine cycle. The computation is carried out through several cycles until the solution becomes repeatable and independent of the initial conditions. The model was described in details in ref. [3], [7], [10] and [11].

Modelling of processes in a screw engine, based on differential rate equations for mass and energy, ensures in principle a reproduction of the the engine thermodynamics and fluid flow. However, its ability to mimic a real process depends strongly on how faithfully the model accounts for accompanied phenomena and effects, which have to be modelled in an approximate manner. Among these most important are the fluid and energy losses through the clearances between the rotors and casing, as well as between the main and gate rotors themselves, heat transfer between the fluid and the environment, fluid friction losses, real properties of working fluid, phase changes and interphase transfer in the case of two-phase flows. It is the modelling of these effects, which distinguishes the modelling approaches and level of empiricism adopted by various authors. In some cases, most of the effects are accounted for by integral empirical correlations, using similarity arguments and dimensional analysis, and obtained on the basis of the bench test of a specific engine - a privilege of only few major screw engine manufacturers - and curve fitting through the data. However, although these correlations may be tuned to account well for the considered effects in a particular engine, their generality and extrapolation to a new situation, with a purpose of predicting the engine behaviour, may be highly questionable. By adopting a more general approach based on the differential treatment of the mass and energy balance for an elementary control volume for an increment in the rotation angle, it is possible to define most of the phenomena mentioned also in a differential form for the local rotation angle, and integrate all equations simultaneously over the cycle. In such a way the accompanying effects are treated also in a differential form. For example, the clearance flow leakage, $\dot{m}_l$, can be expressed in terms of local variables at a particular position in the engine,

$$\dot{m}_l = \mu_l w_l \rho_l A_l = \mu_l \gamma_l A_l \tag{1}$$

where $\gamma_l = w_l \rho_l$ is the local mass velocity of the leaking fluid, $A_l = l\,\delta$ is the clearance ('gap') cross-sectional area, and μ_l is the 'discharge' coefficient. The leakage gas velocity follows from the differential momentum equation, accounting for the fluid-wall friction

$$w_l\, dw_l + \frac{dp}{\rho} + f\,\frac{w_l^2}{2}\,\frac{dx}{D_e} = 0 \tag{2}$$

where $f(Re, Ma, \Psi)$ is the friction coefficient dependent on local Reynolds and Mach numbers, as well as factor Ψ, which accounts for the shape of the clearance gap. All variables can be defined locally at a position in the engine and in function of the rotational angle. For the elementary control volume the effects of leakage is then accounted for by adding and subtracting corresponding local leakage gains and losses in mass and energy. The model is sufficiently flexible to allow not only the distinction between different types of clearances (leading and trailing rotor tip - housing, interlobe clearance, rotor front - housing), but also a variable clearance gap as well as different discharge coefficients (allowing for different and variable shapes) of each clearance type.

Likewise, the working fluid friction losses can be defined also locally in terms of the local friction factor, local fluid velocity (related to tip speed) and density, and elementary friction area.

The same approach was adopted for modelling the oil (or other fluid) injection and its effects of gas cooling. Assuming that the oil injection nozzle has known characteristics so that the mean Sauter diameter can be *apriori* defined, the local heat transfer between the oil droplets and gas yields the expression for the rate of change of the droplet temperature

$$\frac{dT_o}{d\alpha} = \frac{h_o\, A_o\,(T_{gas} - T_o)}{m_o\, c_o\, \omega} \tag{3}$$

where index 'o' denotes oil, A_o are m_o the droplet area and mass respectively, c_o is the oil specific heat, ω is the rotor angular velocity, and h_o is the heat transfer coefficient between the droplets and gas, evaluated from the formula $Nu = 2 + 0.6 Re_o^{0.6} Pr^{0.33}$.

By using the local fluid properties for the particular position in the engine at a local rotation angle α, supplied from separate subprogrammes, and incorporating the local values of leakage flows, friction losses and oil-gas interaction, one can integrate the model to yield the evolution of all thermodynamic and fluid properties in function of the rotation angle. The obtained data can further be integrated over the cycle to obtain bulk properties defining the engine output and performance efficiency.

The models of these and other effects must be verified either separately or within the the computer programme as a whole, before it can be employed for optimization or prediction purposes. The present model used for the optimization example described in this paper was verified by comparison with laboratory measurements of all important instant and bulk properties in an air screw compressor, yielding satisfactory agreement with experiments.

As seen from the above few examples, even the simple differential equations for each phenomena described, influence the engine performances in a complex manner so that the optimum geometry and working conditions can not be established even when considering only each effect individually. In reality all effects are present and often they exercise opposing effect, emphasizing further the need for an optimization with respect to all important parameters, be it on a bench stand in laboratory, or by computation modelling. A case in point are the leakage and friction losses, which, particularly at high tip speeds are the major energy losses in the screw engines. Pressure drops at the inlet and discharge, as well as friction losses during the compression or expansion process are proportional to the square of the tip speed. Since the fluid flow rate is also roughly proportional to the tip speed, specific energy losses due to friction per unit flow rate of the working fluid increase approximately linearly with the tip speed. On the other hand, gas leakage is not very sensitive to the tip speed, but relative leakage per unit working fluid flow rate is inversely proportional to the tip speed. It is obvious that there must be an optimum tip speed which will result in the minimum sum of these two major energy losses per unit flow rate, or unit power of the engine. Of course, this is an oversimplified picture: there are many other factors influencing the process which can be accounted for only by complex optimization. It should be also pointed out that the optimization criteria should be defined for each particular case, depending on the engine application, exploitation regime specific cost-effectiveness or technological requirements, and other factors.

Profile Optimization

There are several general criteria for screw profiles optimization, which are valid irrespective of the engine type and duty. An efficient screw engine is expected to ensure the highest possible fluid flow rates for the given engine size and rotor speed, which implies as large as possible fluid flow cross-sectional area. Of course, the maximum delivery per unit size or weight of the engine must be accompanied by optimum power utilization - the minimum for a compressor and maximum for an expander - which implies a maximum efficiency of the energy interchange between the fluid and the engine. This means that unavoidable fluid and energy losses must be kept at minimum. Since the fluid leakage is the major loss in screw engines, one of the most important requirements is to achieve smallest possible clearances and the shortest possible sealing lines. All these requirements must be fulfilled in such a way, as not to impair the machining of the rotors by milling or hobbing with available machine tools, nor to jeopardize the desired accuracy and surface quality.

Complex geometry of the screw engines brings in the play a number of geometry parameters. Although in principle a general multi-variable optimization method can be applied to yield the optimum combination for the defined criteria, it is often more rational to adopt *a priori* certain parameters and restrict the optimization to only few of those, which influence the engine performances in complex manner, not known in advance. For example, the Lysholm combination of cycloides on the high pressure side, ensuring the rotor contact up to the moment when they touch the housing and reducing thus far the blow-hole area to a minimum, has hardly any alternative. Minor variations may be introduced by shortening or extending the radial line approaching the gate rotor rolling circle. The lobe contact at the low pressure side should also be as long as possible to ensure a longer clearance and consequently a smaller interlobe leakage, but also to yield a shorter sealing line. Both requirements are satisfied when the same circle is employed for the contact line between the main and gate lobes. Unfortunately, in this case the lobes are not rolling, but sliding, what may be inconvenient if liquid is being processed. For this reason the circles must be displaced eccentrically, meaning that one of them must be a cycloide. For example, the SRM patented profile had one of the circle centered eccentrically on the gate rotor, whereas the main rotor lobes of newer design consist of two or more circular arches. In the search for a new profile, we have adopted the above described features as a starting point, reducing the number of free parameters to only few. In the present case the following geometry variables were selected to vary free within a prescribed domain: θ_0, r, r_z and r_0 (Fig. 1) In such a case, the optimization procedure can be reduced to a simple variant search by considering series of discrete values of each of the variables, which are subjected to the optimization. An example of such analysis is discussed later in the text.

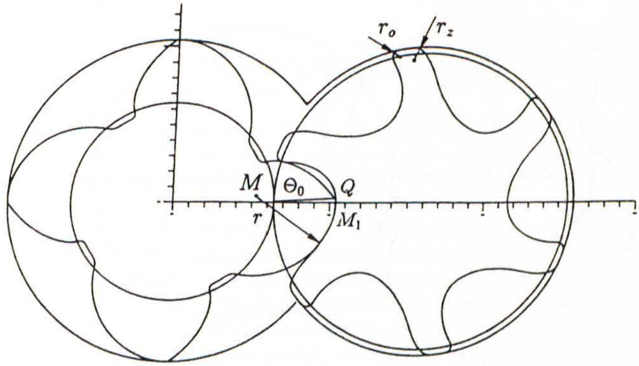

Fig. 1. Compressor rotor pair with new lobe profiles: the optimization parameters are denoted

In the present work we employed the SCORPATH code (Screw Compressor Optimal Rotor Profiling and THermodynamics) which provides facilities for generating new profiles on the basis of pre-specified set of lobe arches (segments) in a general form, and for reproducing the thermodynamic process in the compressor on the basis of the mathematical model in a differential form. The computer package contains also the packages for calculating the thermodynamic and fluid properties of a number of different fluids, including most of the new refrigerants.

Although the computer code can be applied to any type of screw engines, and to a variety of fluids, in the present example we consider a compressor, which is more sensitive to the design details than expander. A dry air compressor seems a suitable choice since the process is close to adiabatic with a large isentrope exponent, κ, close to 1.4. A '102 mm' compressor with axes distance of 80 mm with a modest tip speed of 50 m/s was selected to serve as a basis for the optimization. Typical inlet conditions, p_0 = 1.0 b and T_0 = 300 K were adopted to comply with the usual compressor practice. The discharge pressure was p_d = 3.5 b, corresponding to the built-in volume ratio of 2. As an example we consider only one parameter, the gate rotor tip radius r_0. All other geometry and operating parameters were kept constant. This radius is known to affect directly the cross-sectional area. In the case of asymmetric profiles, when the Lysholm cycloide pair is used to define the lobe profiles on the high pressure side, the gate tip radius r_0 is the major parameter influencing the blow-hole area, which is commonly regarded as the weakest point in the lobe profile design. In fact the size of the blow-hole area is almost directly proportional to the gate tip radius r_0 and in order to reduce the blow-hole, r_0 should be made as small as possible. Ideally, $r_0 = 0$ should be equal to zero (or even 'negative'), which would theoretically eliminate the blow-hole area, but every reduction in r_0 decreases the fluid flow cross-sectional area and the flow rate. Besides, the rotors with extremely small values of r_0 are difficult to manufacture and to maintain in the compressor exploitation. There are other indirect effects of r_0 such as on the flow leakages, but these effects are difficult to quantify. Fig. 2 shows the two rotor pairs with two different tip radii, $r_0 = 0.5$ and 3. A blow-up of the rotor tips are shown in Fig. 3.

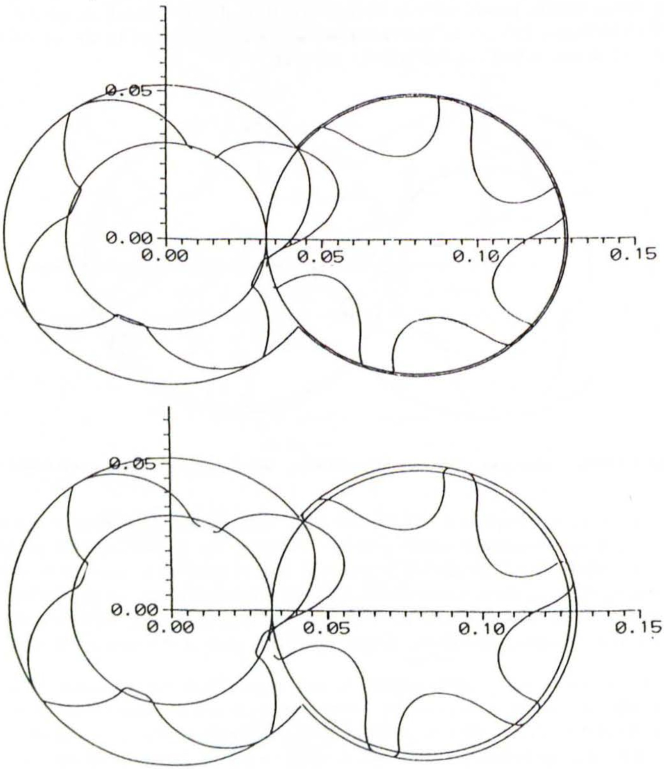

Fig. 2. Rotor pairs with new profiles for two different gate rotor tip radii

Fig. 3. A close look at the rotor tips with two different tip radii, showing the blow-hole area

Because of complex effects on the engine performance, the optimum value of r_0, expected to be a small positive value, can be found only by optimizing the profile with respected to the chosen optimization criteria. Irrespective of the criteria, however, $r_0 = 0$ can serve as the lower bound of the optimization domain. The other bound can be freely chosen. In the present study r_0 was adopted to vary between 0 and 3 mm. A series of discrete values of r_0 were selected and fed into the SCORPATH preprocessor, which generated corresponding series of different profiles. For each of the profiles the thermodynamic process was simulated yielding all relevant process parameters. Since the optimum performances vary depending on the engine application, specific requirement, exploitation regime, all major parameters were plotted in terms of r_0. In the present study we seek for the minimum specific power, but also other performance characteristics have been considered. The figures which follow show the results of computations. Fig. 4 and Fig. 5 show the compressor power and delivery rate, both increasing monotonically by approximately 20% with an increase in r_0 from 0.5 mm to 3 mm. Of course, due to the losses, the two curves are not directly proportional, meaning that their ratio should exhibit an extremum, corresponding to the optimum solution. This is illustrated in the Fig. 4, showing the compressor power per unit delivery rate ('unit power'). The ordinate scale has been blown up to emphasize the variation of the unit power, although, admittedly, the changes between the minimum and the largest value in the considered range of variation of r_0 is less than one percent. However, it should be noted that the computations yielded a expected smooth variation in a consistent manner with a very pronounced minimum at $r_0 = 1.6$ mm, indicating a complex effect of the tip radius r_0 on the engine efficiency.

Because the engine performances can be valued on the basis of different optimization criteria, two other efficiency parameters, the adiabatic and volumetric efficiency, are plotted in function of r_0 in Figures 5 and 6, respectively. The adiabatic efficiency has its peak at $r_0 = 1.65$ mm, close to the optimum of unit power, whereas the volumetric efficiency reaches its maximum at $r_0 = 1.1$ mm. This indicates that the optimum value of the tip radius r_0 is not uniquely defined and its choice requires a closer definition of the optimization criteria, or a subjective analysis with account for some other factors. In any case, it is obvious that the zero r_0 is far from the optimum.

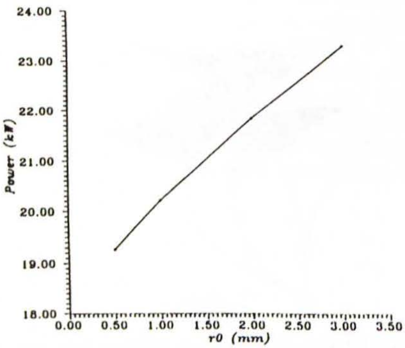

Fig. 4. Variation of compressor power with the tip radius

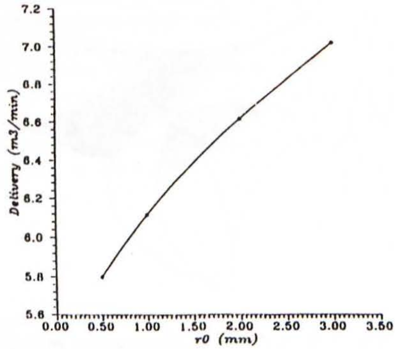

Fig. 5. Variation of compressor delivery rate with the tip radius

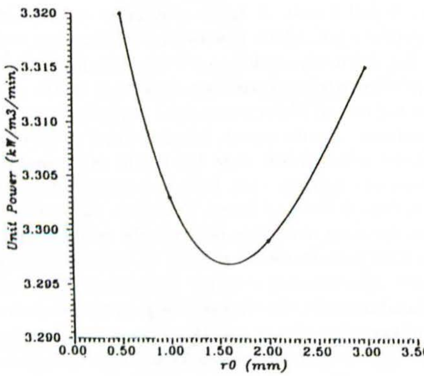

Fig. 6. Variation of unit power with the tip radius

A further insight in the effect of the considered geometrical parameter can be obtained from Fig. 7, where the computed $p - V$ diagrams for four different values of r_0 are plotted together. Figure clearly indicates the increase in the unit work, and consequently of the power for the same rotor speed, with an increase in r_0, but apart from that, the diagram gives no indication on the process quality.

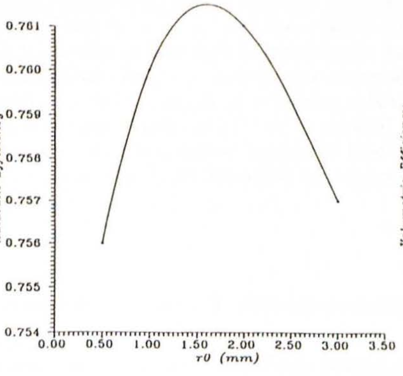

Fig. 7. Variation of the compressor adiabatic efficiency with the tip radius

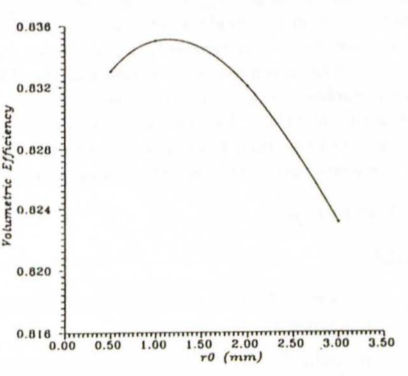

Fig. 8. Variation of the compressor volumetric efficiency with the tip radius

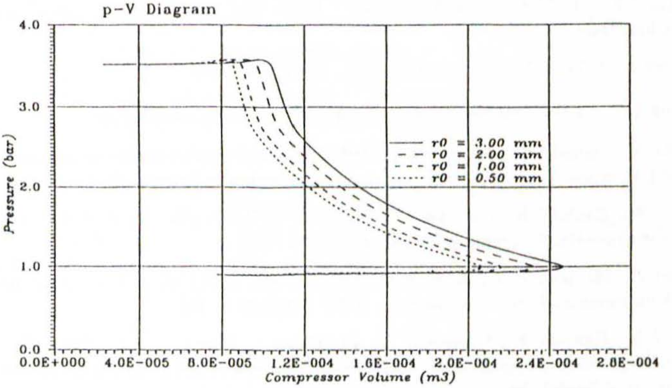

Fig. 9. Indicator diagrams for four values of the gate rotor tip radius

Conclusions

The computational package SCORPATH has been applied to demonstrate the possibility of optimizing the design and working parameters of screw engine rotor profiles, as well as for further improvement of the design and engine performances. The computer package, developed by authors, provides the general specification of the lobe segments in terms of several key parameters, which can generate various lobe shapes, enabling the computation of instant cross-sectional area and working volume in term of rotation angle. The package contains also the mathematical model of the thermodynamic process, as well as models of associated processes encountered in real engines, such as variable fluid leakages, oil flooding or other fluid injection, heat losses to the environment, friction losses and other effects, all expressed in differential form in term of an increment of the rotation angle. The paper presented an example of the application of the package to the analysis of the effects of variable gate rotor tip radius, as well as to finding its optimum value. The tip radius was chosen because of its complex effects upon the screw engine performances, but the computer package could be applied to multi-variable optimization of the engine geometry and its working parameters for the define optimization criteria.

Acknowledgement

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