Optimisation Methods for Rotors of Twin-Screw Compressors

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Abstract

A general reflection concerning the modelling of machines using several abstraction levels is presented. For an exemplary twin-screw compressor this results in a representation of the rotor profile by means of bézier segments. Beginning with arbitrary initial profiles, evolution strategies show their basic suitability by optimising single criteria such as the scoop area. Depending on the initial form, only local optima are found. Using an extended offspring generation scheme to modify the optimisation process, search behaviour can be improved.

1   Introduction

Designing any type of machine leads inevitably to the question of the "optimised" construction parameters. As a rule this can be estimated by the performance with regard to user requirements. The main problem is to define acceptable performance and evaluate a possible solution with low calculation complexity. Usually there are many different criteria to be considered. Thus an appropriate optimisation method has to be able to either represent different criteria within one utility function or to treat several criteria in parallelly. If furthermore the optimisation is supposed to run partially or completely automatically, there arises the problem to derive an adequate simulation model.

This problem will be investigated below using a dry running twin-screw compressor as a representative example. It is intended to show how to evolve a computer-based model starting with user requirements and passing through several levels of abstraction.

The emphasis of the article is an investigation into the optimisation of the two-dimensional rotor profile, which possesses high priority for the twin-screw compressor's functionality. The representation of the profile by parametrical spline functions serves as a new approach compared with the traditional method of combining fixed curve types manually. Finally exemplary optimisations of simple

criteria reveal the aptitude and behaviour of evolutionary strategies as a tool for finding optimised solutions.

2   State of the Art

2.1   Twin-Screw Plants

The design of twin-screw plants and of the twin-screw machine's geometry depend largely upon the planned operational conditions as well as upon the type of gas applied. Usually the design process starts with the calculation of the thermodynamic cycle. Thereafter further calculations are done, based on available geometries and standard machines. Procurable models determine the number of stages, the plant set up and rotational speeds.

The optimisation of a single compressor stage is mainly characterised by the determination of geometric data like the number of lobes, the flank form of the rotor profile, the rotor length, the wrap angle, the position of the guiding edges as well as by operational data such as the rotational speed and - where applicable - the amount of liquid injection. The development of the profile is generally separate from the design of the other construction parameters.

For the determination of these data it is possible to use geometric calculations /1 - 4/ such as the calculation of clearance areas, chamber volumes etc. or more detailed calculations involving FE-Methods in order to analyse both thermodynamic and geometric effects /4 - 8/. In the case of industrial development it is common practise to include much empirical data into the design process of new twin-screw machines.

2.2   Flank Forms

Initially there is the construction of a suitable profile design of the rotors due to its significant influence on the machine's operational behaviour, thermodynamic and geometric properties and on the machining. The relevance of the cross-section results from the operational principle of twin-screw machines, which work without any contacting seals between their working chambers. Varying the flank form and varying the number of lobes influences the geometry and the position of the intermesh clearances, the volume of the working chambers and the utilisation of the construction volume.

Concerning the profile design, the company Svenska Rotor Maskiner succeeded in 1970 in developing a new asymmetrical profile instead of the common circle-arc profile, which led to an efficiency increase of 10% /1/. Buthmann /6/ shows that the

polytropic efficiency[1] can be increased by further modifications of the profile form from η_{pol} = 0,856 to η_{pol} = 0,915. Also in the recent past, further evidence of successful profile improvements can be found in literature like the development of the N-profile by Stosic /9/. These examples demonstrate the potential of fundamental geometry optimisations.

In the past the development of the cross section profile geometry was generally based on the combination of a few basic curve types. Typical applied curve types are lines, circle arcs, evolvents, parabolics, cycloids or equidistant curves to cycloids. Many of the investigated patents deal with improvements of the geometrical, thermodynamic or machining properties. Often the inventions are based on certain curve types or only modify existing profile types.

Due to the capability of modern computer systems, several attempts to convert the geometrical description of profiles to discrete points or parametrical curves have been made. This permits modifications of flank geometries in a more flexible and automated way, in order to meet specific operational demands. Examples can be taken from the literature about general problems of tooth design /10 - 15/.

This article investigates mainly how to use similar approaches for the optimisation of twin-screw rotors. Firstly the investigations are limited to planar rotor profiles. As a speciality it has to be mentioned that the construction of the flanks is provided to run non-interactively but completely automatically by means of a computer program. The only interactive procedure is to determine certain criteria and their corresponding weightings initially or to select suitable solutions from a set of optimised profiles after an optimisation procedure. Thus the user of such an optimisation program does not need any specific knowledge about meshing theory because it is already implemented.

2.3 Optimisations Methods

As an optimisation method an evolution strategy has been chosen. The term evolutionary algorithm (EA) is used to classify stochastic optimisation methods that simulate the process of natural evolution. The origins of EA can be traced back to the late 1950s and, since the 1970s several evolutionary methodologies have been proposed, mainly genetic algorithms, evolution strategies, and evolutionary programming /16, 17/. They all share the common conceptual base of simulating the

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[1] $$\eta_{pol} = \frac{w_f}{w_f + \varphi}$$

with:
w_f specific reversible delivery work
φ specific dissipation

evolution of individual structures via a process of selection and reproduction. New individuals are created using genetic operators such as mutation and crossover. The probability of survival of the newly generated solutions then depends on their fitness, i.e. how well they perform with respect to the given optimisation problem. What makes EAs so attractive from the optimisation point of view is that they do not make much in the way of mathematical demands on the optimisation model. They are 0-order methods, which means that all they need is an objective function to be evaluated - without any derivation. Further, they can handle non-linear problems, defined on discrete, continuous or mixed search spaces, unconstrained or constrained /18/.

The class of the evolution strategies (ESs) has been selected, because they use real-valued vector representations, and are thus best suited to the given problem. **Fig. 1** outlines a typical ES /19/.

```
t = 0;

initialise population P(t);

evaluate P(t);

until (done) {

    t = t +1;

    parent_selection P(t);

    recombine P(t);

    mutate P(t);

    evaluate P(t);

    survive P(t);

}
```

Fig. 1:   The evolution strategy algorithm

A common notation to characterise an ES has been introduced by Rechenberg /20/. He summarises the main strategy parameters in the form: (μ , λ), with μ being the number of parents and λ being the number of offspring, separated by the selection operator. The ',' represents a strategy, where the parents of the following generation are only selected from the offspring of the current generation. Another commonly used strategy is the '+' selection, where parents can survive and thus the parents of the following generation are selected from the offspring and parents of the current generation together.

2.4   Multiple Criteria Decision

Solving multi-objective scientific or engineering problems is in general a very difficult goal. One usually faces the difficulty of conflicting objectives and limited scope for

decision making. As a result high dimensionality extensive computational resources are needed.

Whereas single-objective optimisation problems may have a unique optimal solution, multi-objective optimisation Problems (MOPs) usually present a possibly infinite set of solutions /21/. Optimisation problems involving multiple, conflicting objectives have in the past often been approached by aggregating the objectives into a scalar function and solving the resulting single-objective optimisation problem, whereas here the goal is to find a set of optimal trade-offs, the so-called Pareto-optimal set.

A multi-objective search space is partially ordered in the sense that two arbitrary solutions are related to each other in two possible ways: either one dominates the other or neither dominates /16/.

Definition 1: *Let us consider, without loss of generality, a multi-objective optimisation problem with m decision variables (parameters) and n objectives:*

$$
\begin{aligned}
Minimize \quad & \vec{y} = f(\vec{x}) = (f_1(\vec{x}), \ldots, f_n(\vec{x})) \\
where \quad & \vec{x} = (x_1, \ldots, x_m) \in X \\
& \vec{y} = (y_1, \ldots y_n) \in Y
\end{aligned}
\qquad \text{eq. (1)}
$$

and where $\vec{x}$ is called decision vector*, X parameter space, $\vec{y}$ objective vector, and Y objective space. A decision vector $\vec{a} \in X$ is said to dominate a decision vector $\vec{b} \in X$ (also written as $\vec{a} \prec \vec{b}$) if and only if*

$$
\begin{aligned}
\forall i \in \{1, \ldots, n\}: \quad & f_i(\vec{a}) \leq f_i(\vec{b}) \quad \wedge \\
\exists j \in \{1, \ldots, n\}: \quad & f_i(\vec{a}) < f_i(\vec{b})
\end{aligned}
\qquad \text{eq. (2)}
$$

With this, nondominated and Pareto-optimal solutions can be defined:

Definition 2: *Let $\vec{a} \in X$ be an arbitrary decision vector:*

1. *The decision vector $\vec{a}$ is said to be* nondominated *regarding a set $X' \subseteq X$ if and only if there is no vector in X' which dominates $\vec{a}$.*

2. *The decision vector $\vec{a}$ is* Pareto-optimal *if and only if $\vec{a}$ is nondominated regarding X.*

Pareto-optimal decision vectors cannot be improved in connection with any objective without causing a degradation in at least one other objective; they represent in this context globally optimal solutions.

EAs have become established as *the* method for exploring the Pareto-optimal front in MOPs that are too complex to be solved by exact methods, like linear programming and gradient search /16/. The two main reasons for this are their ability to handle

extremely large search spaces and the convenience to approximate the whole Pareto-optimal front in one single run.

3 Abstraction Levels of the Modelling Procedure

Because of the complexity of general construction problems it is not possible to present a consistent method for the abstraction of any desired machine or component. Thus the method briefly described below has to be seen as a suggestion, which would need to be adapted to other problems analogously. More detailed publications concerning this topic are intended.

3.1 Function

Starting with maximum detailed knowledge of the machine's functionality and its physical mode of operation, it is necessary to determine first which are the most important user demands (inputs) and the most important construction parameters (outputs). Analysing the interdependencies between input und output parameters helps to obtain an order of priorities for the construction parameters and to decouple groups of parameters. This makes it possible to simplify the optimisation problem into sub problems that can be optimised separately.

From an abstract point of view the user of a twin-screw compressor requires a thermodynamic flow, which leads to a modification of the fluid's energy content, mainly its pressure energy. Therefore an order of priorities for the inputs can be determined. They are completely different from the machine's design parameters, **table 1.**

User Demands (Inputs), Sorted by Decreasing Priority	Construction Parameters (Outputs), Unsorted
Pressure ratio	Rotational speed
Volume flow	Number of lobes
Fluid type	Tooth pitch or axis distance
Inlet pressure	Profile form
Inlet temperature	Length / diameter ratio
Purchase and operational expenses	Wrap angle
	Geometry and position of the guiding edges

Table 1:   Results of a simplified functional analysis

In order to ascertain priorities for the outputs it is useful to estimate the effect of varying a single output on the performance of the input requirements. This can be realised within a weighted matrix, containing all possible and evaluated interdependencies. Weighting them according to the input priorities and adding together the weighted interdependencies of each output parameter leads to an order of priorities for all outputs. It is inevitable that an evaluation of the interdependencies

is influenced by subjective ratings or has to be roughly estimated. In this case the matrix has to be revised when more details are revealed.

Applied to a twin-screw compressor the above method reveals two main results:

o Due to the mandatory requirement of developing suitable working chambers and due to their substantial influence on thermodynamical and mechanical properties, the geometric data of the rotor profile becomes the most important set of parameters.

o In a first step it seems logical to optimise the two-dimensional profile independent of three-dimensional data such as rotor length and wrap angle.

3.2 Key Figures

Once a solution is determined, it is necessary to evaluate its performance, which results from its physical properties. Usually technical optimisation problems are too complex to be solved analytically. Therefore iterative working methods are inevitable, but they have to evaluate a huge amount of different solutions on their way to an optimum. As a consequence it is not possible to simulate the solution's performance exactly, but instead to find simple criteria with low calculation costs, which correspond with physical tendencies. Usually this leads to a focus on criteria, which can be calculated from geometrical data.

Criterion	Name	Physical Meaning	Definition
K_1	Scoop area per male rotor rotation	As this area delivers the fluid it characterises the geometrical exploitation of the construction size.	$$K_1 = A_{MA} + A_{FE} \cdot \frac{z_{MA}}{z_{FE}}$$ eq. (3)
K_2	Referred polar moment of inertia	K_2 characterises the resistance to deflections for a constant rotor length, wrap angle and pressure load. In order to consider the worst case, the minimum male and female values are selected.	$$K_2 = \min\left(\frac{I_{p,MA}}{D_{MA}} ; \frac{I_{p,FE}}{D_{FE}}\right)$$ eq. (4)
K_3	Blow hole height	The distance between the contact line and the cusp represents one dimension of the blow hole, which influences the internal leakage rate and thus the thermodynamic performance	$K_3 = h_B$ eq. (5)

Table 2:   Exemplary criteria for profile optimisation
Symbols:
A   Tooth space area of all teeth,
z   Number of lobes,
I   Moment of inertia
D   Outer diameter
h   Height

Indices:
MA   Male rotor
FE   Female rotor
B   Blow hole
p   Polar

Evaluation criteria of twin-screw compressors can be divided into several main topics:

- o geometry,
- o mechanics,
- o thermodynamics,
- o machining.

Corresponding key figures can be found in /1, 2, 4, 22/ or have to be determined by the user. A useful method is the Π-theorem /23/, which converts single key figures into dimensionless key figures by analysing their units.

In order to optimise a two-dimensional rotor profile, three simple exemplary criteria for a constant axis distance were chosen /24/, **table 2**.

These three criteria are in fact incomplete for the purpose of obtaining suitable profiles, but they allow the basic properties of the optimisation method to be tested. Currently more than 10 criteria in all have been implemented. Future investigations will focus on the implementation of additional criteria and their suitability for a competitive optimisation.

Using several criteria the problem of determining their priority occurs once again. Consequently it is necessary to estimate their influence on user demands in the same way as the influence of the construction parameters. This leads to the following priorities:

- o scoop area,
- o referred polar moment of inertia,
- o blow hole height.

This article focuses on the optimisation of single criteria, but multiple criteria optimisations are planned for the future.

3.3 Geometry

When a machine is modelled, it may be necessary to depict the geometry of mechanical components. It is worth investigating the possibilities of a geometrical data processing and to define the corresponding geometrical requirements. Geometrical abstraction exercises an important influence on the quality of the solution, the dimension of the mathematical search space and on the calculation costs in general. /25 - 27/ offer helpful basics concerning this topic.

Returning to the sample machine, it is necessary to find a mathematical description for the rotors of twin-screw compressors, starting with the two-dimensional profiles of the male and the female rotor.

In the first instance there are three self-evident possibilities, which are shown in **table 3**, with their varying characteristics.

Owing to the intention of an automated optimisation, the spline method was selected. The next decision concerns the spline type. In order to simplify the mathematical treatment, concerning the calculation of different geometrical data and the verification of some boundary conditions, the choice was made for a combination of parametrical quadratic bézier segments, **fig. 2**, which can also be converted into a quadratic b-spline. The starting point and the end point are part of the curve. The connection of the boundary points and the control point represents the tangent directions at the beginning and the end of the curve respectively. For initial investigations it was accepted that the approximation behaviour of this spline type might be inaccurate for certain meshing situations of rotor profiles. On the other hand the treatment and certain geometrical calculations are less complex with that type of curve. More details about this topic are planned in future publications.

Geometrical Approach	Advantage	Disadvantage
Combinations of different analytical curve sections	o The possibility of the exact reproduction of traditional profiles o Applicability of basic meshing conditions o Existence of analytical solutions for the conjugate flanks for certain curve types o Low dimension of the search space	o Complication of automated computing due to the treatment of different curve types o Partially higher calculation costs for geometrical data such as curvature, normal vectors etc. depending on the curve type
A set of discrete flank points	o Independence from certain curve types	o No applicability of the basic meshing conditions o No calculation of certain geometric data such as curvature, normal vectors etc. o High dimension of the search space
A spline with a consistent section type	o Applicability of basic meshing conditions o Low dimension of the search space o No distinction between different curve types o Partially adaptable standard solutions for geometrical problems in the literature	o Different approximation quality of analytical curve types o Higher calculation costs when simple curve types are approximated

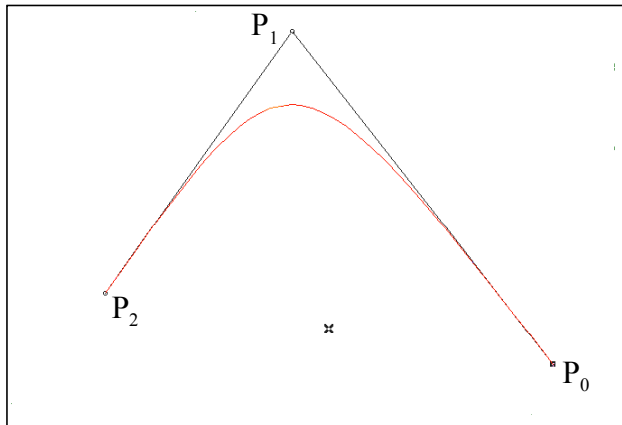*Table 3:   Properties of different geometrical approaches for rotor profiles*

Fig. 2: *Bézier segment (curve) and its control polygon (lines) for the generation of profile flanks*
P_0 Starting point
P_1 Control point
P_2 End point

3.4 Data structure

For an automated optimisation it is necessary to provide a data structure, which lives up to the demands of a machine-specific representation on the one hand, and which guarantees quick access to the optimisation methods on the other hand.

Because of the complexity of the problem an object-orientated programming language is an essential requirement. This provides a hierarchical data structure, where deriving data from basic classes and adding specific properties can lead to the construction of more complex data classes.

The implementation of the profile optimisation is realised by C++ and has a modular structure in order to offer specific libraries. The system can be divided into the sections below:

- o geometrical representations,
- o profile representation and calculation,
- o optimisation,
- o visualisation,
- o files,
- o common functions.

The design of the data classes has an important influence on the readability of the source code and the system's ease of maintenance. In order to obtain a proper class design, not only is knowledge of the programming language needed but also a lot of experience concerning the development of logical structures /28/. **Fig. 3** gives an example of the structure of the profile class, which was applied for the optimisations

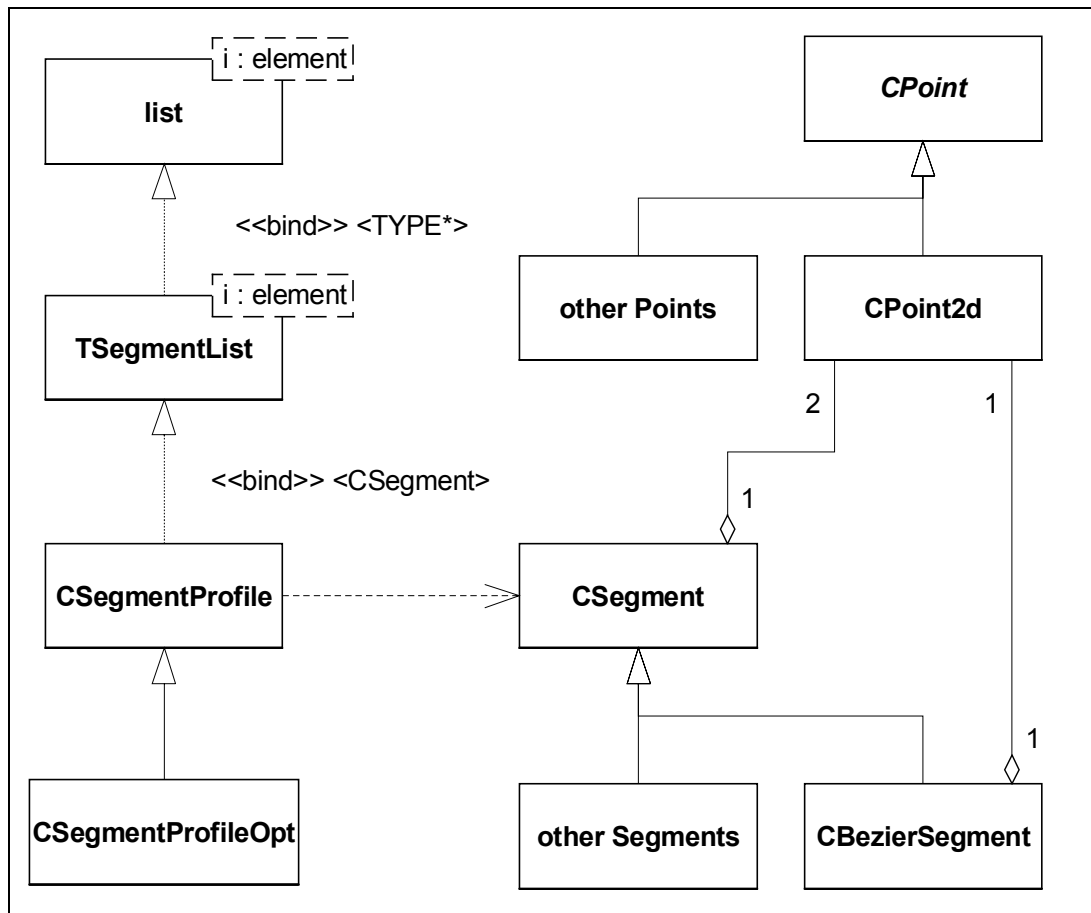in **chapter 4.3**. It uses classes of different sections such as geometry, profile and optimisation.

Fig. 3: *Simplified data structure for the optimisation of twin-screw profiles*

CPoint:	*Abstract class for the representation of points*
CPoint2d:	*Class for two-dimensional points with both Cartesian and polar coordinates*
CSegment:	*Class for the representation of segments with a starting and an end point*
CBezierSegment:	*Class for the representation of bézier segments with a starting, a control and an end point*
list:	*List of a standard template library*
TSegmentList:	*List of pointers to segment elements providing geometrical functions*
CSegmentProfile:	*Class with basic profile data and basic segment types*
CSegmentProfileOpt	*Class with additional data and functions for an optimisation with bézier segments*

4   Optimisation

4.1   Optimisation Method

During optimisation the generation of feasible offspring turns out to be very difficult. The classical (μ, λ)-ES /29/ does not cope with the given constraints. During an initial phase no problems occur and it is possible to create all the λ desired offspring. After a while more and more descendants of the population are not feasible any more due

to one or more of the mechanical constraints. This leads to a point where none of the children falls into the feasible region thus terminating the optimisation.

To overcome this problem two techniques have been tested, as a penalty function method /18, 30/ the global competitive ranking /31/ and an extended offspring generation scheme. The first technique aims to handle both feasible and infeasible offspring whereas the second one tries to keep the offspring valid.

4.1.1 Global Competitive Ranking

The purpose of this method is to use information from both feasible and infeasible individuals. In this scheme, an individual i is ranked by comparing it against all other members of the population. This has to be done for both the objective and penalty functions. Afterwards a unified ranking is set up. For this it is essential to strike the right balance between objective and penalty functions.

The drawback of this approach is that for the given problem a population that has left the feasible region is never able to re-enter it. Unfortunately in every trial the infeasible region was entered after a few hundred generations.

4.1.2 Extended Offspring Generation Scheme

To be able to provide enough feasible individuals in the population, a new generation scheme has been deployed. In contrast to standard ES more attempts per child to create a valid offspring are undertaken. The number of attempts can be chosen by the variable $attMax$.

The procedure then works in the following way: A first attempt to generate a feasible offspring is carried out. If this is successful the procedure turns toward the next child. In case of failure the remaining attempts are carried out. Again, if an attempt is successful, the next child will be processed. If it is not possible to create a valid child within $attMax$ attempts, the next step depends on the number i of the actual offspring. If the child is one of the first $\lambda - \mu$, then it is left invalid and will die out in the next generation. If it is instead one of the last μ, it has to be guaranteed that there will be at least μ valid offspring to build up the next generation. So if the number of offspring remaining to be generated is less then the number of the parents minus the actually valid offspring, then the current child has to be replaced by the corresponding parent. In other words parent $\mu_{\lambda-i+1}$ has to be selected if $\lambda - i < \mu - ofs$ with ofs being the number of actually feasible offspring. The whole algorithm is shown in **fig. 4**.

With this generation scheme a test has been done to see how many attempts are reasonable for the given problem. The parameters were chosen as follows: (3,10) - ES, 2500 generations, $attMax = 1, 10, 20, 50, 100, 200, 500$ each 5 runs.

```
    ofs = 0;
  for i = 1 to λ {
     att = 0;
     do {
        generate(λ_i);
        att = att + 1;
     }
     while (is_not_feasible(λ_i) & att < attMax)
     if (is_feasible(λ_i))
        ofs = ofs + 1;
     else
        if (λ − i < μ − ofs) {
           λ_i = μ_{λ−i+1};
        }
  }
}
```

Fig. 4: **The Extended Offspring Generation Scheme with**

$\mu :$ **Parents**

$\lambda :$ **Offspring**

$ofs :$ **Feasible offspring**

$att :$ **Attempts**

$attMax :$ **Maximum attempts**

$attMax$	Averaged valid offspring per generation
1	0.93
10	2.35
20	2.13
50	3.65
100	4.17
200	4.97
500	6.34

Table 4: *Averaged number of valid offspring per generation for different numbers of attempts*

What can be thought on looking at **table 4** is that $attMax$ has to lie between 20 and 50 if there are going to be on average enough valid offspring to build up the next generation. Further, up to a value of 200 for $attMax$, not even half the offspring are feasible on average. But that is only half the truth, as can be seen in **fig. 5** and **fig. 6** respectively.

The vital point is that it is relatively easy to create offspring during the initial phase even with few attempts, whereas later fewer and fewer feasible children are generated. So the mean value over all generations is higher than the mean value at the end of the optimisation, and thus the stated value might be not very meaningful.

Another observation is that after 500 attempts the optimisation succeeds in recovering, which means that it is possible to come from few valid offspring back again to a larger population.

To sum up, the extended offspring generation scheme is an instrument to provide the ES with the required number of offspring. It has an impact on the selection scheme by modifying it to span the whole range from a pure comma strategy to a pure plus strategy, covering all shades between these two extremes and doing this dynamically.
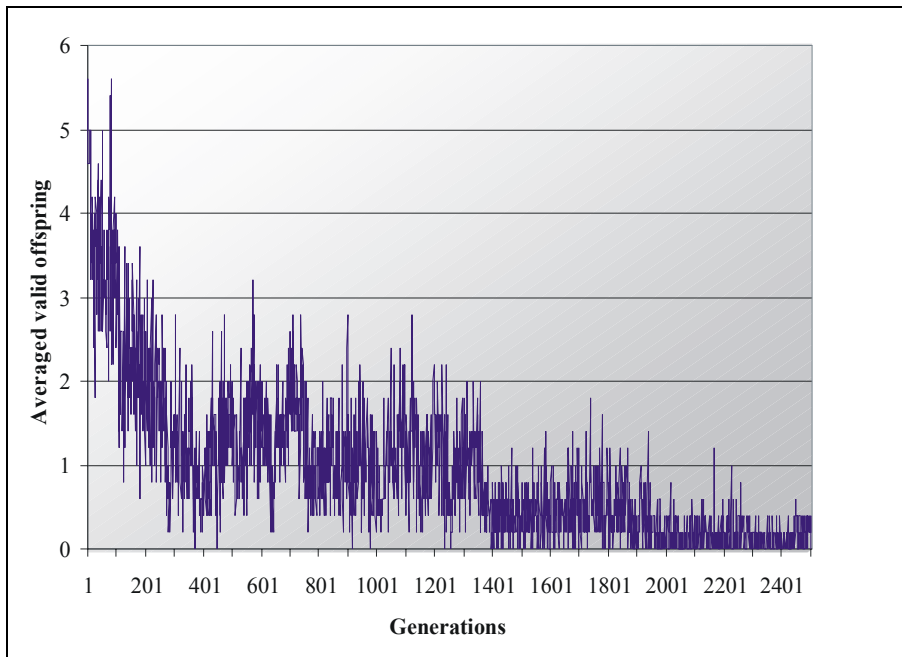

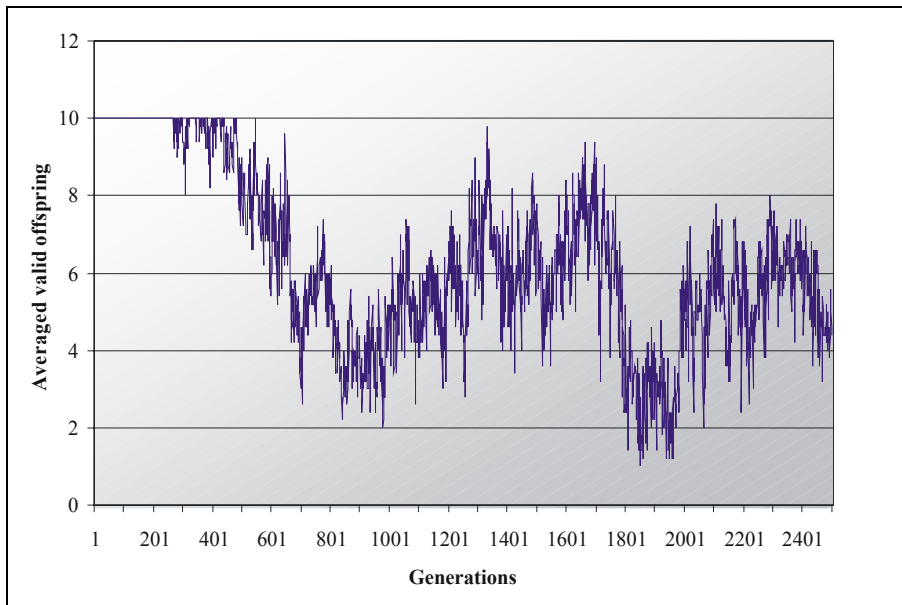Fig. 5: The development of the offspring for $attMax = 1$.

Fig. 6: The development of the offspring for $attMax = 500$.

4.1.3 Optimisation Parameter

The optimisation strategy can be configured with several parameters. The core parameters are:

- o μ: the number of the parents in the population
- o λ: the number of the offspring to be created
- o selection operator:
    determines whether parents can survive or not (**chapter 2.3**)
- o the number of generations

If recombination (i.e. sexual reproduction) is desired, it can be controlled by:

- o ρ: the number of parents that are used to generate an offspring
- o the mixing scheme: discrete / intermediary

For the mutation of the segments radii, angles, and gradients of points have to be processed. For each, values have to be provided for:

- o lower limit of the mutation step size
- o upper limit of the mutation step size
- o initial mutation step size

As an asymmetrical mutation scheme has been implemented /32/, apart from this, for each mutation (i.e. for the radii, the angles, and the gradients) there also have to be given:

- o lower limit of the asymmetry
- o upper limit of the asymmetry
- o initial value for the asymmetry

Last, for the extended creation scheme:

- o *attMax*:
    states how many attempts are carried out to create a valid offspring (**chapter 4.1.2**)

4.2   Automated Geometry Synthesis

The class CSegmentProfile represents the two dimensional profile of one rotor for twin-screw compressors, containing the flank form of one lobe and additional data such as the number of lobes, the root, the crown and the roll circle. It can contain different segment types, but for the optimisation bézier segments were used exclusively. Furthermore the tangents of adjacent segments are coincident at their

common boundary point. Therefore a variety of different flank forms can be produced with a relatively low number of segments.

4.2.4 Flank Generator

As evolutionary algorithms are processed, the production of random profiles is desirable. In order to obtain profiles which observe certain basic conditions a "flank generator" was implemented /24/.

The flank generator can be initialised with the data below:

- o number of lobes of the generator and the conjugate profile,
- o root circle of the generator,
- o crown circle of the generator,
- o roll circle of the generator,
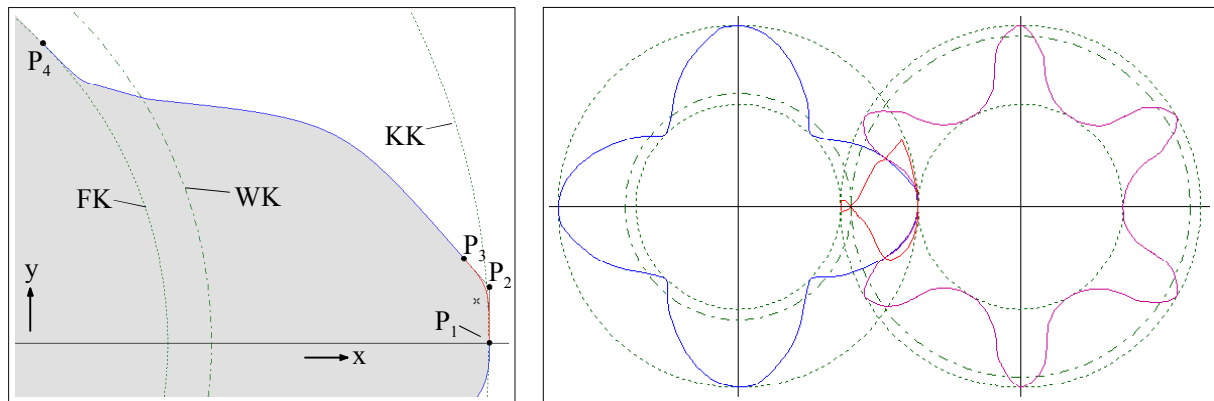- o number of segments of the generator.

Fig. 7: *Tooth flank built by a random generator (left) and the following calculation of the conjugate rotor and the mesh line data (right)*

P$_{1...3}$ *Base points of the first segment*
P$_4$ *Prescribed dedendum*
FK *Root circle*
WK *Roll circle*
KK *Crown circle*

The most important conditions, which are verified during the segment-wise construction process, are:

- o tangential starting at the crown circle, and tangential ending at the root circle,
- o segment courses between root and crown circle as well as within the lobe angle,
- o smooth connection of all segments,
- o creation of a closed contact line (required to build up working chambers).

Fig. 7 illustrates the result of a single flank's automated construction.

4.2.5 Flank Manipulator

Since an initial flank form is produced, it is necessary to manipulate its geometry randomly and to filter out better solutions according to the above optimisation algorithm, **chapter 4.1**.

Three manipulation operators have been deployed to change the profile geometry (e.g. with reference to point P_3 in **fig. 7)**:

- o   variation of a base point's polar radius,
- o   variation of a base point's polar angle,
- o   variation of a base point's tangential direction.

After each manipulation all boundary conditions have to be checked, and non-valid profiles are excluded out or the evaluation is corrected by means of penalty functions.

4.3   Exemplary Profile Optimisations

Former investigations focussed on optimisation of the single criteria in **table 2** /24/. **Fig. 8** summarises the main results on their way from the corresponding initial geometry to the improved modifications. Since only single criteria where optimised the results are not competitive for realistic twin-screw compressors, which have to take into account several criteria simultaneously (scoop area, blow hole area, length of the contact line, small deflections etc).

Current research work concentrates on the influence of the initial form on the result and on the reproducibility of the method. Depending on the criterion and the boundary conditions, it is quite likely to get bagged down in local optima. It is part of this examination to start with the same geometry several times on the one hand and to start with explicitly different geometries on the other hand. After the optimisation process the differences between the results are analysed comparatively .

The investigations are not yet finished, so only a few results can be presented at present. Four initial profiles have been realised as shown in **fig. 9,** with radically different specifications. All profiles fulfil the basic meshing conditions and have a closed contact line. Every form was applied to the scoop area, criterion K_1.

For profiles a), b), and c) only preliminary results are available, because of short optimisation runs due to time limitations. With profile d) extended tests have been done, because it seems to be the most awkward profile.

Several test runs with rotor a) have been undertaken, all leading to more or less the same result. At the end there was always a quadrate-like shape with rounded edges, **fig. 9**. Some constraints prevent the plain sides from caving in. Further investigations have to be done to be able to ascertain the reasons for this behaviour.
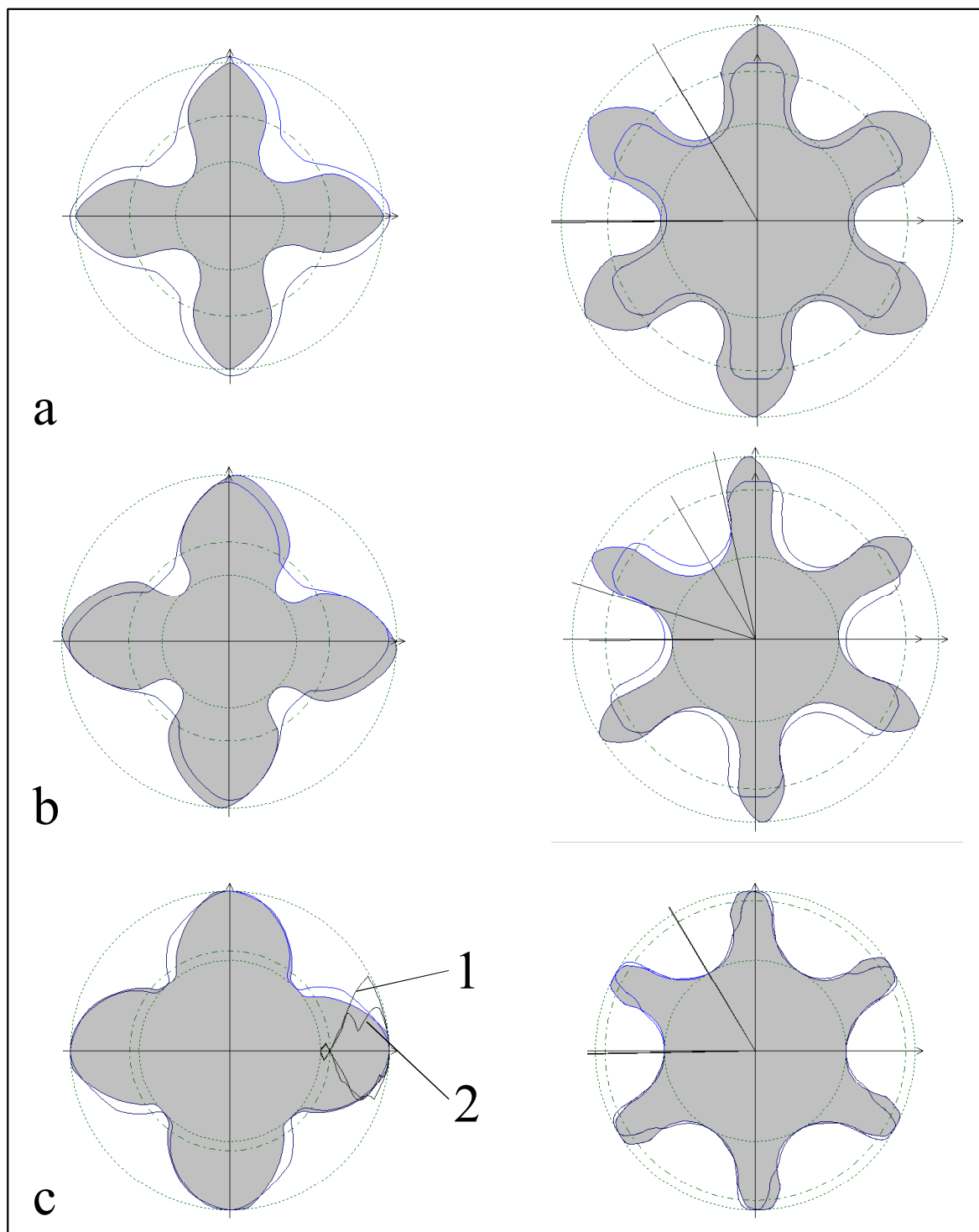
Fig. 8: *Comparison of initial profiles (hollow) and optimised profiles (shaded grey) for different criteria and fixed axis distance*
Left: Male rotor
Right: Female rotor
a)   Maximised scoop area (root and crown circle not fixed)
b)   Maximised referred polar moment of inertia (root and crown circle not fixed)
c)   Minimised blow hole height with mesh line of the optimised profile (1) and of the initial profile (2) (root and crown circle fixed)
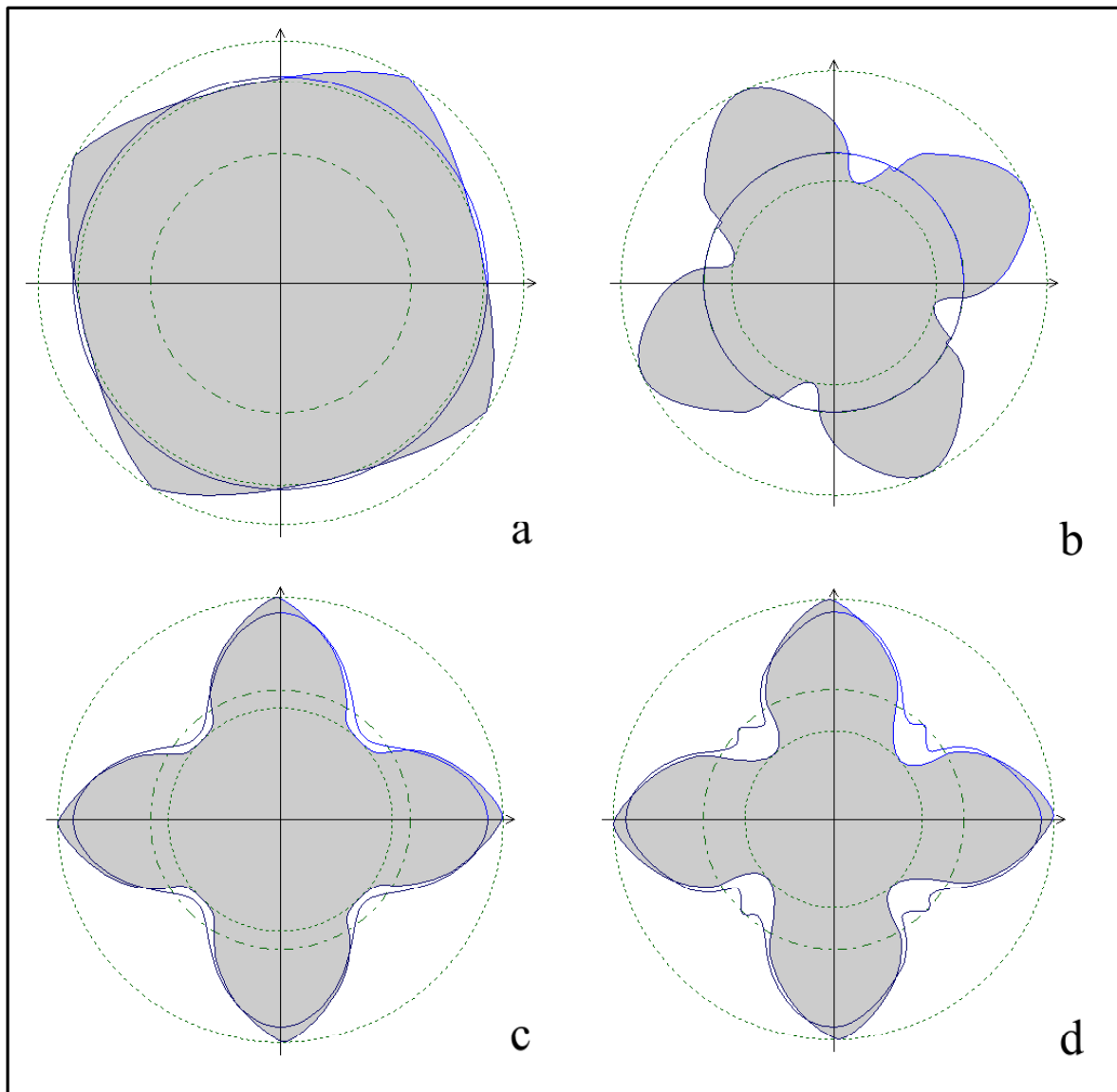
Fig. 9: *Different initial profiles (hollow) for the determination of their influence on the optimisation result (shaded grey), each consisting of 10 bézier segments*
Initial profiles:
a)   Approximated crown circle
b)   Approximated roll circle
c)   Symmetrical regular profile
d)   Asymmetrical, extremely abnormal random profile

Because of the results achieved with rotor a) a different circular profile has been introduced. Whilst profile a) can be seen as an inflated version of c) and d) with all points lying on the crown circle, profile b) represents another extreme. Here all points lie on the roll circle. The results that have been obtained with this profile meet expectations better, **fig. 9**. The rotor shown was achieved after 5,700 generations. Chances are high that the bulge on one flank will be eliminated later as could be seen during the optimisation of profile d).

The optimisation of profile c) progressed in an unexpected way. It was just not the easiest case. On the contrary, this profile was the one where the feasibility of the offspring turned out to be the hardest problem. Several runs led to the result that after a few thousand generations no valid offspring could be generated. The reasons for this are not yet known. Nevertheless the optimisation is tending to advance in the right direction, as shown in **fig. 9** for a rotor of generation 1,500.

The most interesting challenge could be seen in optimising profile d). It was very interesting to check if the optimisation would cope with this extremely abnormal profile. After 20,000 generations a rotor had been generated that has a scoop area of $8.1 \cdot 10^{-3}$ m^2, thus being comparable to the most successful one from former trials, **fig. 8**. What also emerged is that the optimisation is capable of straightening out bulges. All this tends to confirm that the chosen algorithms are suitable for solving the given problem.

5   Perspective

The optimisation procedure presented here has demonstrated its suitability in principle, but does not yet lead to competitive profiles. Furthermore its properties on the way to an optimum have to be investigated more intensively.

Concerning the optimisation method, the points below are planned to be investigated:

- o   influence of the optimising parameters,
- o   implementation of methods for multi-objective optimisation,
- o   more experiments on reproducibility and on initial forms are needed along with other criteria such as the blow hole width and the referred moment of inertia.

In order to obtain more realistic profiles it is projected to

- o   process further criteria, which are already implemented but not yet tested,
- o   vary more profile parameters such as the number of lobes and the number of segments and to
- o   extend the method to the three dimensional design of rotors including rotor length and wrap angle.

Finally a sensitivity analysis will reveal the influence of single parameters and the possibility of decoupling a two dimensional from a three dimensional rotor-design optimisation.

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